THz parametric gain in semiconductor superlattices in the absence of electric domains

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We theoretically show that conditions for THz gain and conditions for formation of destructive electric domains in semiconductor superlattices are fairly different in the case of parametric generation and amplification. Action of an unbiased high-frequency electric field on a superlattice causes a periodic variation of energy and effective mass of miniband electrons. This parametric effect can result in a significant gain at some even harmonic of the pump frequency without formation of electric domains and corruption from pump harmonics.

Electrically dc-biased semiconductor superlattices (SLs) [1], which operate in conditions of single-miniband transport regime, exhibit static negative differential conductivity (NDC) due to existence of Bloch oscillations [1]. It has been predicted that under the NDC conditions and for a homogeneous distribution of electric field inside nanostructure, SL should provide a strong broadband gain with a maximum at THz frequencies [2, 3]. Therefore, the dc biased SL can potentially be an active element of miniature, tunable and room-temperature operating source of THz radiation (Bloch oscillator). However, a choice of operation point at the part of voltage-current characteristic with negative slope makes the SL unstable against formation of high-field domains [2, 4]. The electric domains destroy the high-frequency Bloch gain in a long SL. In order to stabilize the electric field inside SL two new types of nanostructure design have been recently introduced: Super-SL comprised of a stack of short SLs [5] and lateral surface SL shunted by another SL [6].

In recent letter [7], we suggested to use a microwave unbiased electric field (pump frequency ω) instead of dc bias to achieve a high-frequency gain in domainless SL devices operating at a positive slope of voltage-current characteristic. We theoretically proved the existence of strong negative absorption of a sub-THz probe field with the frequency $\omega_1 = n\omega$ (n is odd number). Such high-frequency gain in SL was termed parametric. However, because SL is a strongly nonlinear medium, odd harmonics of the pump field, $n\omega$, are simultaneously generated [8]. As a rule, the effect of generated harmonics blur out the weaker effect of THz parametric gain. This problem is well-known for the parametric amplification at microwave frequencies in Josephson junctions, which also have strong nonlinearity [9].

In this respect earlier work of Pavlovich [10] merits notice. Using approach similar to [2], he calculated the coefficient of absorption of a weak probe field (ω_1) in SL subjected to a strong pure ac field of arbitrary frequency ω for the case of integer or half-integer ratio of ω_1/ω. The value of relative phase ϕ between the pump and probe

fields was fixed to be zero. This interesting work has not received much attention so far [11]. Nevertheless, simple calculations employing the original Pavlovich formula with $\phi = 0$ show that in principle gain at $\omega_1 = n\omega$ (where $n \geq 4$ is *even number*) can exist for very strong pump fields belonging to THz frequency range $\omega\tau \simeq 1$ (the characteristic relaxation time of electrons τ is about 100 fs at room temperature). Amplification at even n will not be corrupted from harmonics of the pump, because none of even harmonics can be generated in the unbiased symmetric SL. Thus, the parametric amplification at even harmonics is worthy of further investigation. Here the following main questions arise: What is the physical mechanism for amplification? What is the role of the relative phase? Is it possible to avoid a formation of the destructive electric domains within this scheme?

In this letter, we show that a significant parametric gain at an even harmonic arises in a superlattice due to a periodic variation of effective masses of miniband electrons with the frequency that is twice pump frequency or its even harmonic. We demonstrate that the parametric gain can occur for the amplitudes and relative phases of the pump and probe fields for which destructive electric domains are not formed inside the superlattice. In particular, we find that the parametric gain for small probe fields always exists in the absence of NDC, if the pump frequency belongs to the low-frequency part of THz range $\omega\tau \lesssim 0.45$.

We suppose that the total electric field $E(t)$ acting on SL electrons is the sum of the pump $E_0 \cos(\omega t)$ and the probe $E_1 \cos(\omega_1 t + \phi)$ (Fig. 1). Dynamics of the electrons belonging to a single miniband is well described by the semiclassical balance equations [8, 12]

$$
\begin{aligned}
\dot{V} &= qE(t)/m(\varepsilon) - \gamma_v V, \\
\dot{\varepsilon} &= qE(t)V - \gamma_\varepsilon(\varepsilon - \varepsilon_0),
\end{aligned}
\tag{1}
$$

where $V(t)$ and $\varepsilon(t)$ are the electron velocity and energy averaged over the time-dependent distribution function satisfying the Boltzmann equation, $q(= -e)$ is the electron charge, ε_0 is the equilibrium energy of carriers, γ_v and γ_ε are respectively the relaxation constants of the average velocity and energy. The dependence of the electron effective mass $m(\varepsilon)$ on the energy is

$$
m(\varepsilon) = \frac{m_0}{1 - 2\varepsilon/\Delta},
$$

Figure 1: (color online) Schematic representation of SL device. Parametric gain arises at an even harmonic $\omega_1 = n\omega$ (green online) of the unbiased pump field (red online).

where $m_0 = 2\hbar^2/\Delta a^2$ is the effective mass at the bottom of the miniband (a is the SL periods and Δ is the miniband width). Stationary solutions ($t \to \infty$) of balance equations (1) have the remarkable symmetry properties, which allow to conclude that in the unbiased case $V(t)$ can have only odd and $\varepsilon(t)$ only even harmonics of ω [13]. Therefore, under the action of pump field both electron energy and effective mass vary with the frequencies $\omega_\varepsilon = 2n\omega$ ($n = 1, 2, 3, \ldots$). For SL placed into an external circuit (resonant cavity) with the resonant frequency ω_1 the parametric resonance condition becomes $\omega_1 = \omega_\varepsilon/2$. In terms of pump and probe fields, one should expect parametric gain for a probe field with the frequency $\omega_1 = \omega_\varepsilon/2 = n\omega$ (Fig. 1). The pump field should be strong enough in order a negative work done on the miniband electrons in the parametric resonance can overcome a loss due to free-carrier absorption. Parametric interactions always depend on the value of relative phase ϕ; gain is expected only for some ranges of ϕ.

Here we are interested in the phase-dependent gain at even harmonics, i.e. for $n = 2, 4, \ldots$. We numerically solve the balance equations for different values of the pump E_0 and probe E_1 amplitudes, relative phases ϕ, for several values of the product $\omega\tau$ ($\tau \equiv (\gamma_v \gamma_\varepsilon)^{-1/2}$) and for several ratios $\gamma_\varepsilon/\gamma_v$. We calculate the phase-dependent absorption of the probe field in SL as $A = \langle V(t)\cos(\omega_1 t + \phi)\rangle$, where time-averaging $\langle\ldots\rangle$ is performed over the period of stationary motion $2\pi/\omega$. Gain corresponds to a negative value of A. We found that gain arises for at least one value of ϕ if E_0 exceeds some threshold value $E_{th,n}$, which is specific for every n and also depends on $\omega\tau$.

On the other hand, destructive electric domains will not be formed inside SL if (i) dependence of the averaged current $\langle V(t)\rangle$ on dc bias E_{dc} has a positive slope at the working point and (ii) the frequency of the pump field is larger than the inverse characteristic time of domain formation τ_{dom}^{-1} (τ_{dom} is of the order of the dielectric relaxation time) [7]. The last condition, $\omega\tau_{dom} > 1$, can be satisfied for typical SLs with doping $N \lesssim 10^{16}$ cm^{-3} if the pump frequency $\omega/2\pi \gtrsim 100$ GHz ($\omega\tau \gtrsim 0.1$) [7]. To check fulfilment of the first condition we add an infinitesimal bias E_{dc} to the pump field and numerically find a sign of the derivative $d\langle V\rangle/dE_{dc}$ at the working point $E_{dc} = 0$. NDC and formation of domains correspond to the negative derivative.

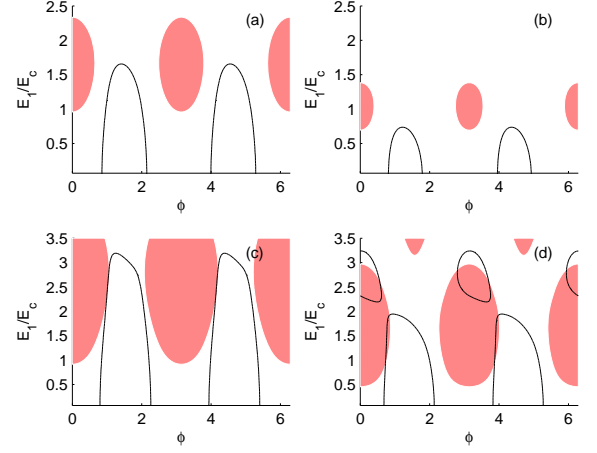Figures 2 and 3 show the regions of gain and domain

Figure 2: (color online) Regions of gain ($A < 0$) and regions of domain formation (red online) for the low-frequency pump $\omega\tau = 0.1$ and for (a) $n = 2$, $E_0/E_c = 3$; (b) $n = 4$, $E_0/E_c = 3$; (c) $n = 2$, $E_0/E_c = 5$; (d) $n = 4$, $E_0/E_c = 5$. Everywhere $\gamma_\varepsilon/\gamma_v = 1$.

formation in the plane ϕE_1 for different pump amplitudes E_0 and respectively for the microwave ($\omega\tau = 0.1$) and THz ($\omega\tau \simeq 1$) pump frequencies. The field amplitudes are scaled to the Esaki-Tsu critical field $E_c = \hbar/qa\tau$ [1, 12]. For small $\omega\tau$ the regions of gain at second (Fig. 2a) and at fourth (Fig. 2b) harmonics are well separated from the regions of domains for all amplitudes of probe field. In these cases the values of E_0 only slightly exceed the threshold amplitudes $E_{th,2}/E_c = 1.7$ and $E_{th,4}/E_c = 2.6$. With a further increase of E_0 these regions of gain and domains start to overlap slightly for large probe amplitudes (Figs. 2c,d). Moreover, for relatively high pump amplitudes or/and for gain at higher harmonics new second-row regions of gain and domains arise for high probe amplitudes (Fig. 2d). As a rule, the second-row gain regions strongly overlap with the regions of domains (Fig. 2d).

It is worth to notice that in the quasistatic limit $\omega\tau, \omega_1\tau \ll 1$ a series of runs demonstrated no overlapping of the lowest NDC and gain loci in the plane ϕE_1 for $n = 2, 4, 6$ and reasonable pump strengths $E_{th,n} \leq E_0 \lesssim 5E_c$. Therefore, the domainless parametric amplification of *microwave fields* in SLs under *microwave pump* is achievable in lightly doped SLs ($N \ll 10^{16}$ cm^{-3}) with additionally satisfied condition $\omega\tau_{dom} > 1$ (note: $\tau_{dom} \propto m_0/N$ [7]).

Now we turn to the case of THz pump $\omega\tau \simeq 1$ (Fig. 3). Complete separation of NDC and gain regions is also possible in this case as evident from Fig. 3a. This figure also provides a good illustration for the existence of a small-signal gain at $\phi = 0$ and $n = 4$ in agreement with the original Pavlovich formula [10]. We underline that it requires a very strong pump. However, gain is still achievable for lower pump amplitudes; in this case
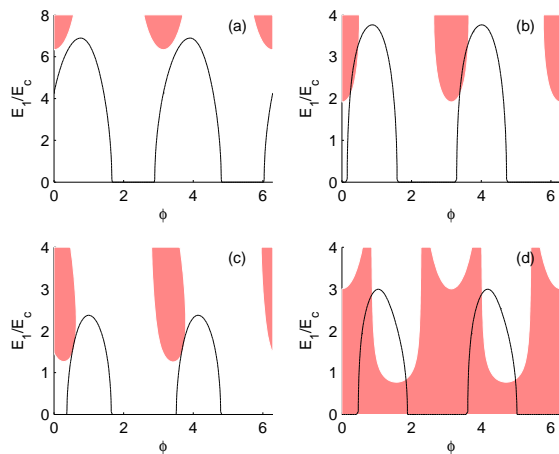
Figure 3: (color online) Regions of gain and domains (red online) for the high-frequency pump fields: (a) $n = 4$, $E_0/E_c = 8$, $\omega\tau = 1.2$, $\gamma_\varepsilon/\gamma_v = 1$; (b) $n = 2$, $E_0/E_c = 4$, $\omega\tau = 1.05$, $\gamma_\varepsilon/\gamma_v = 1$; (c) same as (a) but for $\gamma_\varepsilon/\gamma_v = 0.1$; (d) $n = 2$, $E_0/E_c = 4$, $\omega\tau = 0.75$, $\gamma_\varepsilon/\gamma_v = 1$.

the range of ϕ supporting gain is shifted from the point $\phi = 0$ (Figs. 3b,c). Importantly, we observed no sufficient qualitative changes in the pictures for the choice of a two different relaxation constants $\gamma_v \neq \gamma_\varepsilon$ instead of a single constant (e.g., cf. Figs. 3b and c). Note that in the quasistatic limit locations of NDC and gain in the plane ϕE_1 are completely independent on the ratio $\gamma_\varepsilon/\gamma_v$.

In all figures presented so far no NDC exists for all ϕ in the limit of weak probe $E_1 \to 0$. Situation is changed drastically if the pump field itself can cause NDC for all ϕ (Fig. 3d). It happens if $\omega\tau \gtrsim 0.45$ and $\alpha \equiv \frac{qaE_0}{\hbar\omega} = \frac{E_0}{E_c\omega\tau}$ is close to one of the Bessel roots: $J_0(\alpha) = 0$ (Fig. 3d corresponds to $\alpha = 5.33$ that is near the second root 5.52). Obviously, the situation when the pump parameters correspond to the Bessel roots should be avoided in order to reach domainless gain.

In the limit of small probe field, the phase-dependent gain always has a maximum at some optimal phase ϕ_{opt}. We found that the values of ϕ_{opt} are located *at the centers of gain ϕ-intervals* for all $\omega\tau$. In the quasistatic limit

$\phi_{opt} = \pi/2$ and $3\pi/2$ [14]. For parameters used to plot Figs. 2a, 2b, 2d, 3a, 3b, and for $a = 6$ nm, $\Delta = 60$ meV, $\tau = 200$ fs, $N = 10^{16}$ cm^{-3}, the values of small-signal gain calculated at $\phi = \phi_{opt}$ and at room temperature are respectively 35.3, 10.6, 23.3, 4, and 34.9 cm^{-1}. The magnitude of parametric gain in SLs is no less than the estimated Bloch gain [3].

Oscillator based on the parametric gain in SL should include an external resonant cavity (circuit) tuned to $n\omega$. Our preliminary simulations of the model, which include the balance equations (1) together with the resonant cavity equation, demonstrate that in conditions of parametric resonance SL selects $\phi = \phi_{opt}$ among other phases of initial field fluctuations in the cavity. Although for $\omega\tau \simeq 1$ the computed phase of a large field in the cavity was found to deviate already sufficiently from ϕ_{opt}, it is still, as a rule, distinctly different from the phase values supporting formation of domains. Therefore, *for the case of oscillator* partial overlapping of the lowest gain and domain regions, like those shown in Figs. 2c, 2d, 3b, will pose *no obstacle to the domainless operation* of device.

SL also can be used as an active element of regenerative parametric amplifier. In this case, the phase of pump and the phase of an external amplified signal determine the value of phase difference ϕ. For operation of SL parametric amplifier it is important to choose the pump amplitude E_0 and the range of signal amplitudes E_1 in such a way that the corresponding region of gain would be well separated from domains in the plane ϕE_1.

In summary, solving balance equations we have shown that use of ac pump field instead of dc bias in room-temperature superlattice devices allow to get a significant parametric gain for THz frequencies in the absence of electric domains. In conclusion, we would like to notice that following [15] balance equations of the same functional form as Eq. (1) can well describe a THz response of hot electrons in dilute nitride Ga(AsN) alloys. Therefore, we suggest to explore the dilute nitride alloys as an active media for THz parametric devices.

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